1. Give an example of 3 events A, B, C which are pairwise independent but not independent. Hint: find an example where whether C occurs is completely determined if we know whether A occurred and whether B occurred, but completely undetermined if we know only one of these things.

Consider two fair, independent coin tosses, and

let A be the event that the first toss is Heads,

B be the event that the second toss is Heads,

and C be the event that the two tosses have the same result.

Then A, B, C are dependent since P(A n B n C) = P(A n B) = P(A)P(B)=1/4 ≠ 1/8 = P(A)P(B)P(C),

but they are pairwise independent: A and B are independent by definition;

A and C are independent since P(A n C) = P(A n B)=1/4 = P(A)P(C), and similarly B and C are independent.

1. A bag contains one marble which is either green or blue, with equal probabilities. A green marble is put in the bag (so there are 2 marbles now), and then a random marble is taken out. The marble taken out is green. What is the probability that the remaining marble is also green?

Let A be the event that the initial marble is green,

B be the event that the removed marble is green,

and C be the event that the remaining marble is green.

We need to find P(C|B).

There are several ways to find this; one natural way is to condition on whether the initial marble is green: P(C|B) = P(C|B,A)P(A|B) + P(C|B,A^c )P(A^c |B) = 1P(A|B)+0P(A^c |B).

To find P(A|B), use Bayes’ Rule:

P(A|B) = P(B|A) P(A) / P(B)

= (1/2) / P(B|A)P(A) + P(B|A^c) P(A^c)

= (1/2 ) / (1/2+1/4 )

= 2 /3 .

So P(C|B)=2/3.